$$I_0=4\frac{1-\exp[-\nu\epsilon\ln 2\nu]}{\epsilon\ln 2\nu},\qquad (A7)$$

$$I_1=-4\frac{I_0-4\nu\exp[-\nu\epsilon\ln 2\nu]}{\epsilon\ln 2\nu},\qquad (A8)$$

$$I_2=-8\frac{I_1+8\nu^2\exp[-\nu\epsilon\ln 2\nu]}{\epsilon\ln 2\nu}.\qquad (A9)$$

If we note that, for $s>t_D$, $[\nu/(\nu+1)]^{1/2}\simeq 1$, we obtain from Eq. (2),

$$R_1^{1}(\nu)=\frac{4\pi^2}{\sigma_t}\frac{a_I}{a_R^2+a_I^2},\qquad (A10)$$

where we have used the relation $\sigma_t=4\pi^2\beta(0)$, which can be obtained by combining Eq. (14) with the optical theorem. To simplify our result further we could drop all the exponential terms in Eqs. (A7), (A8), and (A9). This does not affect the final result very much and corresponds to setting the lower limit -4ν equal to $-\infty$ in Eq. (21). Such a procedure is not unreasonable, since $4\nu\gtrsim 80$ with $s>t_D$, and $A^I(t,s)$ is important only in the region $|t|\lesssim 20$, as we have seen. A further simplification results from the *a posteriori* observation that $a_R^2\ll a_I^2$. If we make all these approximations, we finally obtain

$$R_1^{1}(\nu)\simeq\frac{\pi^2}{\sigma_t}\frac{\nu(\epsilon\ln 2\nu)^2}{\nu\epsilon\ln 2\nu-2}.\qquad (A11)$$

With $\sigma_t=75$ mb this gives $R_1^1(t_D/4)=5.31$. If we did not drop a_I we would get $R_1^1(t_D/4)=5.28$. These two are practically indistinguishable. The latter value was the one actually used in the calculations of Secs. V and VI.

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Three-Meson Model for p-p Scattering and Regge Poles

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By considering a dispersion relation for that amplitude of p-p scattering, whose imaginary part in the forward direction is related to the total cross section, it is shown that the one-meson-exchange model (taking into account independent exchanges of the pion, the ρ-ω vector pair, and an $l=0$, scalar 2π resonance or meson with a mass somewhat greater than two pion masses) and high-energy behavior of the p-p and p-$\bar{p}$ scattering cross sections as given by the Regge pole hypothesis, are consistent with the existing p-p scattering data. In our demonstration the energy range involved is larger than previously used in the demonstration of either of the above two aspects of p-p scattering. Further by considering a dispersion relation and high-energy behavior of another amplitude of p-p scattering, it is shown that the second type of coupling of the Pomeranchuk pole is zero. This reduces the number of unknown parameters in the expression for polarization at high energy.

1. INTRODUCTION

It has been shown by several authors[1–4] that the so called one-meson-exchange model, taking into account independent exchanges of one pion, one η, one ρ, and one ω only, gives an excellent approximation to the experimentally observed nucleon-nucleon scattering. In Refs. 2, 3, and 4 it has been necessary to postulate the existence of an $l=0$ scalar meson or resonance of mass 3 to 4 m_π with a rather large coupling constant with nucleon. So far, there appears to be contradictory evidence on the existence of such a meson or resonance.

The energy involved in the above demonstration of the goodness of the one-meson-exchange model is up to 350 MeV. On the other hand, it has also been shown that high-energy behavior[5] of the nucleon-nucleon scattering amplitude or cross section can be explained in terms of the Pomeranchuk pole, P' trajectory, and

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[1] Riazuddin and M. J. Moravcsik, Phys. Letters **4**, 243 (1963).
[2] R. A. Bryan, C. R. Dismukes, and W. Ramsay, Nucl. Phys. (to be published). See also, R. S. Mckean, Jr., Phys. Rev. **125**, 1399 (1962); and D. B. Lichtenberg, J. S. Kovacs, and H. McManus, Bull. Am. Phys. Soc. **7**, 55 (1962).
[3] A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963).
[4] W. Ramsay, Phys. Rev. **130**, 1552 (1963).
[5] S. D. Drell, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN, 1962* (CERN, Geneva, 1962).

TABLE I. Residues of the pole-term contributions to H_1 from the various mesons acting as intermediate particles in proton-proton scattering. Superscripts 1, 2, 3, 4, and 5 refer to π, η, ρ, ω, and S particles, respectively. The notation is explained in the text. $\Psi^+ = g_1^2 m + g_1 g_2 m_\rho^2/2m$. $\Phi^+ = \frac{1}{2}G_1^2 m + G_1 G_2 m_\omega^2/4m$.

$B^{1,2}$	B^3	B^4	B^5
$\frac{1}{2}f^2 m$	$\frac{1}{2}\left[\Psi^+ + \frac{m_\rho^2}{2m}(g_1+g_2)^2\right]$	$\left[\Phi^+ + \frac{m_\omega^2}{4m}(G_1+G_2)^2\right]$	$g_s^2\frac{m}{4}\left(-2+\frac{2m_s^2}{4m^2}\right)$

the ω and ρ mesons, provided that these mesons are treated as Regge poles.

In Sec. 2 of this article we also demonstrate the goodness of the one-meson-exchange model in such a way that both the above aspects are combined in one relation. In our demonstration we use a one-dimensional dispersion relation of the type given by Goldberger, Nambu, and Oehme,[6] which we write in the forward direction for that amplitude of nucleon-nucleon scattering the impaginary part of which is related to the total cross section by the optical theorem. In the u channel we treat π, η, ρ, and ω as elementary-particle poles. We use our dispersion relation in unsubtracted form and we put in the explicit asymptotic behavior of the amplitude as given by Regge poles in the t channel.[5,7] The dispersion relation is used at zero kinetic energy, giving us a sum rule, which we show is satisfied by putting in known parameters. For this purpose, we also have to take into consideration the contribution of an $l=0$ scalar meson or $J=0^+$ resonance of mass 320 MeV or 2.3 m_π in contrast to 3 to 4 m_π as taken by the authors of Refs. 2, 3, and 4. The ABC phenomenon[8] is also at 320 MeV. Moreover, the coupling constant of this scalar meson with a nucleon is much smaller than that taken in Refs. 2, 3, and 4.

In Sec. 3, by considering a dispersion relation for a particular amplitude, namely, H_4-H_5 (see below and Ref. 1), and its asymptotic behavior in terms of Regge poles, we conclude that the second type of coupling of the Pomeranchuk pole, namely, $A_{2,P}(0)$, is zero. This reduces the numer of unknown parameters in the expression for polarization at high energy for p-p scattering.

2. SUM RULE FOR THE S-WAVE SCATTERING LENGTH

The scattering matrix for nucleon-nucleon scattering is written as

$$T=\frac{m}{E}[H_1+H_2\boldsymbol{\sigma}_1\cdot\mathbf{l}\boldsymbol{\sigma}_2\cdot\mathbf{l}+iH_3(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{l}+H_4\boldsymbol{\sigma}_1\cdot\mathbf{m}\boldsymbol{\sigma}_2\cdot\mathbf{m}+H_5\boldsymbol{\sigma}_1\cdot\mathbf{n}\boldsymbol{\sigma}_2\cdot\mathbf{n}],$$

where $\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$ are unit vectors in the directions of $\mathbf{k}\times\mathbf{k}'$, $\mathbf{k}-\mathbf{k}'$, and $\mathbf{k}+\mathbf{k}'$, respectively; $\mathbf{k}$ and $\mathbf{k}'$ are c.m. momenta in initial and final states.

We consider the p-p scattering amplitude H_1 for which $I=1$. The GNO-type dispersion relation for H_1 in the forward direction is

$$\mathrm{Re}H_1(\nu)=\sum_j\frac{B^j}{2m(\nu-m+m_j^2/2m)}+\frac{1}{\pi}\int_m^\infty\frac{\mathrm{Im}H_1(\nu')}{\nu'-\nu}d\nu'+\frac{1}{\pi}\int_m^\infty\frac{\mathrm{Im}\bar{H}_1(\nu')}{\nu'+\nu}d\nu'+\frac{1}{\pi}\int_{\nu(2m_\pi)}^m\frac{\mathrm{Im}\bar{H}_1(\nu')}{\nu'+\nu}d\nu',\quad(1)$$

where $j=\pi,\eta,\rho,\omega,S$ ($I=0$, J^{0+} meson), $\nu=(2k^2+m^2)/m^2$ is the energy of the incident particle in the lab frame, $\nu(2m_\pi)=2(m_\pi^2/m)-m$, and $\bar{H}_1$ is the corresponding amplitude for p-$\bar{p}$ scattering. The B's are given[1] in Table I. There, f is the renormalized coupling constant between a pseudoscalar meson and nucleon ($f^2=0.08$ for the pion), g_1 and g_2 are the charge and magnetic-moment-coupling constants of the ρ meson, G_1 and G_2 are the same for the ω meson, g_s is the coupling constant of the scalar meson S with the nucleon, m is the nucleon mass, and m_j is the mass of the meson.

Following Igi,[8] we write

$$H_1(\nu)=H_1'(\nu)+F(\nu),\quad(2)$$

where $F(\nu)$ gives a divergent behavior as $\nu\to\infty$ and $H_1'(\nu)$ vanishes at ∞. $F(\nu)$ is determined by P, P', and ω Regge trajectories and is given by

$$F(\nu)=-\frac{1}{4m}\sum_i\frac{A_{1i}^2(0)}{\sin\pi\alpha_i}[P_{\alpha i}(-\nu/m)+P_{\alpha i}(\nu/m)],\quad(3)$$

where the summation is over $i=P,P'$, and ω and $A_{1,i}^2(0)$ are the residues of the respective Regge poles. Now for forward scattering $\alpha_P(0)=1$ and, hence, from (2) and (3),

$$\mathrm{Im}H_1(\nu)=\mathrm{Im}H_1'(\nu)+A_{1,P}^2(0)\nu+\frac{A_{1,P'}^2(0)}{4m}P_{\alpha_{P'}}\left(\frac{\nu}{m}\right)+\frac{A_{1,\omega}^2(0)}{4m}P_{\alpha_\omega}\left(\frac{\nu}{m}\right).\quad(4)$$

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[6] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (N.Y.) 2, 226 (1957); hereafter this will be referred to as GNO.
[7] Y. Hara, Progr. Theoret. Phys. (Kyoto) 28, 1048 (1962).
[8] K. Igi, Phys. Rev. Letters 9, 76 (1962).

Since[5] $A_{1,\omega}{}^2(0)=-A_{1,P'}{}^2(0)$ and

$$\mathrm{Im}H_1(\nu)=\frac{1}{8\pi}(\nu^2-m^2)^{1/2}\sigma_{\mathrm{tot}}(\nu)\,, \tag{5}$$

it follows from (4) that

$$A_{1,P'}{}^2(0)=(m^2/2\pi)\sigma_{\mathrm{tot}}(\infty)\,. \tag{6}$$

Now[5] $\alpha_{P'}(0)=\alpha_\omega(0)=0.5$ as given by the high-energy behavior of the scattering cross section for p-p and p-$\bar{p}$ scattering. Therefore, using one subtracted dispersion relation for $P_{\alpha_{P'}}$ and P_{α_ω}, we get from (1), (4), (5), and (6):

$$\begin{aligned}\mathrm{Re}H_1(\nu)=&\sum_i \frac{B^i}{2m(\nu-m+m_j{}^2/2m)}\\&+\frac{1}{8\pi^2}\int_m^\infty(\nu'^2-m^2)^{1/2}\,d\nu'\left\{\frac{\sigma_{\mathrm{tot}}(\nu')}{\nu'-\nu}+\frac{\bar{\sigma}_{\mathrm{tot}}(\nu')}{\nu'+\nu}\right\}\\&-\frac{\sigma_{\mathrm{tot}}(\infty)}{4\pi^2}\int_m^\infty\frac{\nu'd\nu'}{(\nu'^2-m^2)^{1/2}}-\frac{A_{1,P'}{}^2(0)P_{\alpha_{P'}}(0)}{\sin\pi\alpha_{P'}}\\&-\frac{A_{1,P'}{}^2(0)}{2m}\frac{1}{\pi}\int_m^\infty P_{\alpha_{P'}}\left(\frac{\nu}{m}\right)\frac{d\nu'}{\nu'}\\&+\frac{1}{\pi}\int_{\nu(2m_\pi)}^m\frac{\mathrm{Im}\bar{H}_1(\nu')}{\nu'+\nu}d\nu'\,.\end{aligned} \tag{7}$$

We employ relation (7) at zero kinetic energy, i.e., at $\nu=m$. The left-hand side then gives $\frac{1}{2}a$, where a is the S-wave scattering length. It is well known that the S-wave scattering length is sensitive to the inner part of the nucleon-nucleon potential or, in dispersion theory language, to multiparticle exchange which is partly taken into account by the poly-pion resonances, which we treated separately as pole terms, while the rest of contribution from the multiparticle exchange appears in the last integral of Eq. (7). However, the S-wave scattering length itself satisfies a sort of dispersion relation. In fact, in the zero-range approximation $[\sigma_{\mathrm{tot}}=2\pi a^2/(a^2k^2+1)]$, the integral

$$\frac{1}{8\pi^2}\int_m^\infty(\nu'^2-m^2)^{1/2}\,d\nu'\frac{\sigma_{\mathrm{tot}}(\nu')}{\nu'-m}$$

gives exactly $\frac{1}{2}a$. If we put from 0 to 40 MeV, i.e., for $0\leqslant\nu\leqslant 7m_\pi$,

$$\sigma_{\mathrm{tot}}=2\pi/[k^2+(1/a+\tfrac{1}{2}rk^2)^2]$$

(r being the effective range and $=1.94\ m_\pi{}^{-1}$) as given by effective-range theory, we get

$$\frac{1}{8\pi^2}\int_m^{7m_\pi}(\nu'^2-m^2)^{1/2}\,d\nu'\frac{\sigma_{\mathrm{tot}}(\nu')}{\nu'-m}\approx(\tfrac{1}{2}a-\tfrac{1}{4}m_\pi{}^{-1})\,.$$

The S-wave scattering length is thus cancelled out on both sides of Eq. (7). The residual part of Eq. (7) should not, therefore, be very sensitive to the innermost part of the potential or to the continuum contribution apart from the poly-pion resonances, which we have treated separately as pole terms. For this reason we neglect the last integral in Eq. (7) and get, after cancelling out the S-wave scattering length on both sides,

$$\begin{aligned}0=&\sum_i\frac{B^j}{m_j{}^2}-\tfrac{1}{4}m_\pi{}^{-1}+\frac{1}{8\pi^2}\int_{7m_\pi}^\infty(\nu'^2-m^2)^{1/2}\frac{\sigma_{\mathrm{tot}}(\nu')}{\nu'-m}d\nu'\\&+\frac{1}{8\pi^2}\int_m^\infty(\nu'^2-m^2)^{1/2}\frac{\bar{\sigma}_{\mathrm{tot}}(\nu')}{\nu'+m}d\nu'\\&-\frac{\sigma_{\mathrm{tot}}(\infty)}{4\pi^2}\int_m^\infty\frac{\nu'd\nu'}{(\nu'^2-m^2)^{1/2}}-\frac{A_{1,P'}{}^2(0)P_{\alpha_{P'}}(0)}{\sin\pi\alpha_{P'}}\\&-\frac{A_{1,P'}{}^2(0)}{2m}\frac{1}{\pi}\int_m^\infty P_{\alpha_{P'}}\left(\frac{\nu'}{m}\right)\frac{d\nu'}{\nu'}\,.\end{aligned} \tag{8}$$

Now for kinetic energies greater than 10 GeV or for $\nu\geqslant 71.4\ m_\pi$ we use the formula given by the Regge pole hypothesis[5] and obtain from (8),

$$\begin{aligned}0=&\sum_i\frac{B^j}{m_j{}^2}-\tfrac{1}{4}m_\pi{}^{-1}+\frac{1}{8\pi^2}\int_m^{71.4m_\pi}(\nu'^2-m^2)^{1/2}\frac{\sigma_{\mathrm{tot}}(\nu')}{\nu'-m}d\nu'\\&+\frac{1}{8\pi^2}\int_m^{71.4m_\pi}(\nu'^2-m^2)^{1/2}\frac{\bar{\sigma}_{\mathrm{tot}}(\nu')}{\nu'+m}d\nu'\\&-\frac{\sigma_{\mathrm{tot}}(\infty)}{4\pi^2}\int_m^{71.4m_\pi}\frac{\nu'd\nu'}{(\nu'^2-m^2)^{1/2}}-\frac{A_{1,P'}{}^2(0)}{2m}\frac{P_{\alpha_{P'}}(0)}{\sin\pi\alpha_{P'}}\\&-\frac{A_{1,P'}{}^2(0)}{2m}\frac{1}{\pi}\int_m^{71.4m_\pi}P_{\alpha_{P'}}\left(\frac{\nu'}{m}\right)\frac{d\nu'}{\nu'}\,.\end{aligned} \tag{9}$$

The various terms in (9) are evaluated as follows: To evaluate the integral containing $\sigma_{\mathrm{tot}}(\nu)$, we note that the total cross section for p-p scattering does not show much structure[9,10] in the whole range $(7m_\pi\leqslant\nu'\leqslant 71.4m_\pi)$ and is nearly constant (about 40 mb). Using this fact, the integral is easily evaluated and gives $2m_\pi{}^{-1}$. For $m\leqslant\nu'\leqslant 7.6m_\pi$, the integral involving $\bar{\sigma}_{\mathrm{tot}}(\nu')$ is negligible because of the large denominator $\nu'+m$. For $7.6\ m_\pi\leqslant\nu'\leqslant 29m_\pi$, the function $\bar{\sigma}_{\mathrm{tot}}(\nu')[(\nu'-m)/(\nu'+m)]^{1/2}$ is nearly constant (about $2.36\ m_\pi{}^{-2}$) and hence the integral in this range is easily evaluated. For $29m_\pi\leqslant\nu'\leqslant 71.4m_\pi$, the integral is evaluated by using the experimental data[11] for $\bar{\sigma}_{\mathrm{tot}}(\nu')$ in this range. The total contribution of the integral involving $\bar{\sigma}_{\mathrm{tot}}(\nu')$ is

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[9] M. J. Moravcsik, Ann. Rev. of Nucl. Sci. **10**, 234, (1960).

[10] W. A. Wenzel, in *Proceedings of the 1960 International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960).

[11] A. M. Wetherell, Proc. Phys. Soc. (London) **80**, 63 (1962). C. Cocconi, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN, 1962* (CERN, Geneva, 1962).

TABLE II. Contributions to the dispersion relation (9). For the choice of the coupling constants, see text. Units: m_π^{-1}.

	Lhs	Rhs π	Rhs η	Rhs ρ	Rhs ω	Rhs S	Rhs integral over the cross sections	Rhs P' trajectory	Rhs total
1st choice $G_1^2=15$ $g_s^2=2.4$	0	0.27	0.04	0.62	2.13	−1.51	0.46	−1.97	0
2nd choice $G_1^2=10$ $g_s^2=1.3$	0	0.27	0.04	0.62	1.4	−0.82	0.46	−1.97	0

about 2.31 m_π^{-1}. The integral containing $\sigma_{\rm tot}(\infty)$ gives 3.6 m_π^{-1} when $\sigma_{\rm tot}(\infty)$ is taken to be 40 mb. Using[5,7] $A_{1,P'}^2(0)\approx 15$, and $\alpha_{P'}\approx 0.5$, we have

$$\frac{A_{1,P'}^2(0)}{2m}\frac{P_{\alpha P'}(0)}{\sin\pi\alpha_{P'}}\approx 0.47 m_\pi^{-1}$$

and

$$\frac{A_{1,P'}^2(0)}{2m}\frac{1}{\pi}\int_m^{71.4m_\pi}P_{\alpha P'}\left(\frac{\nu'}{m}\right)\frac{d\nu'}{\nu'}\approx 1.5 m_\pi^{-1}.$$

Collecting the various values, we get from (9)

$$0=\sum_i\frac{B^i}{m_i^2}+0.46m_\pi^{-1}-1.97m_\pi^{-1}, \qquad (10)$$

where 0.46 m_π^{-1} is the contribution from the integrals over the cross sections and −1.97 m_π^{-1} is the contribution from the P' trajectory.

On the right-hand side, the pole terms can be evaluated using Table I and the magnitudes of the coupling constants as indicated by experiments. In particular, g_1 was taken from Sakurai[12] to be $g_1^2=0.6$. From this, following Matthews,[13] we choose $g_2=(\mu_P-\mu_N)m^{-1}g_1$, from which $g_2^2=8.1$. To satisfy relation (10), if we use $G_1^2=15$, we have to take $g_s^2=2.4$; and if we take $G_1^2=10$, we have to use $g_s^2=1.3$. We use $G_2^2=0$, which is consistent with the apparent absence of the ω intermediate state in photoproduction,[14] and with the nucleon form factor calculations.[13,15] For η we use $f^2(2m/m_\eta)^2=2$, which is consistent with other interactions of this particle.[16] The contributions of the pole terms, using the above values of coupling constants, are given in Table II, where we also write separately the contributions from integrals over the cross sections and from the P' trajectory.

One can see from Table II that for the choice $G_1^2=15$, $g_s^2=2.4$, the contribution from the ω pole is the largest and positive, that from the P' trajectory is almost equal to the ω contribution but is negative, and that from the S-particle pole is smaller but comparable and is negative in sign. For the choice $G_1^2=10$, $g_s^2=1.3$, the contribution from the P' trajectory is the largest, that from the ω pole is smaller but comparable and opposite in sign, and that from the S-particle pole is still smaller but significant and of the same sign as the P' contribution. The contributions from ρ and π poles are small and that from the η pole is negligible. These latter contributions are all positive.

To summarize, we conclude that, for the existing data on p-p and p-$\bar{p}$ scattering, the following aspects are mutually consistent: (1) the high-energy behavior of p-p and p-$\bar{p}$ scattering cross sections as given by the Regge pole hypothesis; (2) the 3-meson model of nucleon-nucleon scattering, consisting of the pion, the ρ-ω vector pair, and an $l=0$ scalar 2π resonance or meson with mass somewhat greater than two pion masses. We thus confirm the conclusions drawn by the other authors.[1–4,17] However, the $l=0$ scalar meson which we have used is at 320 MeV where the ABC phenomenon exists and has a smaller coupling constant than that used in Refs. 2, 3, and 4. Also, the energy range involved in our demonstrations is larger than that used in Refs. 1, 2, 3, and 4 and Refs. 5 and 7, where the energy used is below 350 MeV and above 10 GeV, respectively.

3. DISPERSION RELATION FOR THE AMPLITUDE $L=H_4-H_5$

We now consider the amplitude $L=H_4-H_5$ and write it as

$$L(\nu)=L'(\nu)+G(\nu),$$

where $L'(\nu)$ vanishes at ∞ and $G(\nu)$ may diverge as $\nu\to\infty$ and its imaginary part for the forward direction is given by

$$\mathrm{Im}G(\nu)=-\frac{\nu-m}{2m^2}\frac{3A_{1,P}(0)A_{2,P}(0)}{2}-\frac{\nu-m}{4m^2}\left[3A_{1,P'}(0)A_{2,P'}(0)P_{\alpha_{P'}}'\left(\frac{\nu}{m}\right)+3A_{1,\omega}(0)A_{2,\omega}(0)P_{\alpha_\omega}'\left(\frac{\nu}{m}\right)\right]. \qquad (11)$$

[12] J. J. Sakurai, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN, 1962* (CERN, Geneva, 1962).

[13] P. T. Matthews, in *Proceedings of the Aix-en-Provence International Conference on Elementary Particles, 1961* (Centre d'Etudes Nucléaires de Saclay, Seine et Oise, 1961), Vol. 11, p. 87.

[14] M. J. Moravcsik, Phys. Rev. **125**, 734 (1962).

[15] G. R. Bishop, in *Proceedings of the 1962 International Conference on High Energy Physics at CERN, 1962* (CERN, Geneva, 1962).

[16] Riazuddin and Fayyazuddin, Phys. Rev. **129**, 2337 (1963).

[17] H. P. Noyes, Phys. Rev. **130**, 2025 (1963).

Hence the asymptotic behavior of ImL is given by

$$\frac{1}{\nu}\,\mathrm{Im}L \underset{\nu\to\infty}{\longrightarrow} -\frac{3A_{1,P}A_{2,P}(0)}{4m^2} -\frac{3\bar{A}_{1,P'}(0)\bar{A}_{2,P'}(0)}{4m^2}\left(\frac{\nu}{m}\right)^{\alpha_{P'}-1} -\frac{3\bar{A}_{1,\omega}(0)\bar{A}_{2,\omega}(0)}{4m^2}\left(\frac{\nu}{m}\right)^{\alpha_\omega-1}, \quad (12)$$

where

$$\bar{A}_{1,P'}(0)\bar{A}_{2,P'}(0)=A_{1,P'}(0)A_{2,P'}(0)\frac{\Gamma(\frac{1}{2}+\alpha_{P'})}{\Gamma(\alpha_{P'})}\frac{2^{\alpha_{P'}}}{\sqrt{\pi}}.$$

Since $\alpha_{P'}\approx\alpha_\omega\approx 0.05$, it is clear from (12) that Im$L(\nu)$ can go at the most as ν when $\nu\to\infty$. We, therefore, write a GNO-type dispersion relation for $L(\nu)$ in the forward direction with one subtraction at $\nu=m$. Such a dispersion relation is

$$\frac{\mathrm{Re}L(\nu)}{\nu-m}=-\sum_j\frac{L_B{}^j}{m_j{}^2(\nu-m+m_j{}^2/2m)} +\frac{1}{\pi}\int_m^\infty\frac{\mathrm{Im}L(\nu')}{(\nu'-\nu)(\nu'-m)}d\nu' -\frac{1}{\pi}\int_m^\infty\frac{\mathrm{Im}\bar{L}(\nu')}{(\nu'+\nu)(\nu'+m)}d\nu' +\frac{1}{\pi}\int_{-m}^{-\nu(2m_\pi)}\frac{\mathrm{Im}L(\nu')}{(\nu'-\nu)(\nu'-m)}d\nu', \quad (13)$$

where $j=\pi,\ \eta,\ \rho,\ \omega$ (S particle does not contribute in this case) and $L_B{}^j=B_{24}{}^j-B_{25}{}^j$, where $B_{24}{}^j$ and $B_{25}{}^j$ are given in Table I of Ref. 1. We shall neglect the continuum contribution given by the last integral in (13) for reasons given in Ref. 1.

Now if we use expression (12) for the asymptotic behavior of Im$L(\nu)$ and assume that it is dominated by a Pomeranchuk pole, then we get from (13)

$$\frac{\mathrm{Re}L(\nu)}{\nu-m}=-\sum_j\frac{L_B{}^j}{m_j{}^2(\nu-m+m_j{}^2/2m)} +\frac{1}{\pi}\int_m^\infty\frac{\mathrm{Im}L(\nu')}{(\nu'-\nu)(\nu'-m)}d\nu' -\frac{1}{\pi}\int_m^\infty\frac{\mathrm{Im}\bar{L}(\nu')}{(\nu'+\nu)(\nu'+m)}d\nu' +\frac{3A_{1,P}(0)A_{2,P}(0)\nu}{4\pi m(\nu^2-m^2)}. \quad (14)$$

It was shown in Ref. 1 that if we calculate the pole terms by using the coupling constants given in Sec. 2, then the existing p-p scattering experimental data satisfy relation (14) without the last term. Moreover, the last term gives a pole at $\nu=m$. For these reasons we conclude that $A_{2,P}(0)=0$.

The vanishing of $A_{2,P}(0)$ reduces the number of undetermined parameters in the expression for the polarization[7,18] in p-p scattering at high energy. This expression is given by[7]

$$P(t)=\frac{(-t)^{1/2}}{m}\frac{\sigma(p\bar{p})-\sigma(pp)}{\sigma(pp)}\times\left[\frac{4}{\sqrt{3}}\frac{A_{2,P}(0)}{A_{1,P}(0)}-\sqrt{3}\frac{A_{2,P'}(0)}{A_{1,P'}(0)}-\frac{1}{\sqrt{3}}\frac{A_{2,\omega}(0)}{A_{1,\omega}(0)}\right].$$

If $A_{2,P}(0)=0$ as we have concluded, then

$$P(t)=-\frac{(-t)^{1/2}}{m}\frac{\sigma(p\bar{p})-\sigma(pp)}{\sigma(pp)}\times\left[\sqrt{3}\frac{A_{2,P}{}'(0)}{A_{1,P'}(0)}+\frac{1}{\sqrt{3}}\frac{A_{2,\omega}(0)}{A_{1,\omega}(0)}\right].$$

One has some information about $A_{2,\omega}(0)/A_{1,\omega}(0)$; it[7] is $\approx\frac{1}{2}$. So the polarization at high energy is dependent on the single parameter, $A_{2,P'}(0)/A_{1,P'}(0)$.

We are indebted to Professor M. J. Moravcsik for helpful comments.

[18] I. J. Muzinich, Phys. Rev. Letters **9**, 475 (1963).